CONTACT THERMAL RESISTANCE OF MACHINED METAL SURFACES IN VACUO

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The paper presents the results of an experimental investigation of contact thermal resistances of stainless steel and molybdenum samples for the range of compressive loads between 1.5 and $58 \cdot 10^{-5}$ N/m^2 and for an absolute pressure of the surrounding medium of 10^{-4} mm Hg. An equation describing the dependence of the contact heat exchange in vacuo for small compressive loads was derived based on experimental data published in the literature.

Under the high thermal current conditions characteristic of modern technology, the temperature discontinuity at the contact between contiguous surfaces of machine components can attain values of tens and even hundreds of degrees. The largest effects of contact thermal resistances on the temperatures of the elements of high temperature machines is observed in vacuo, i.e., in the absence of thermally conducting gaseous media in the contact region. For an accurate calculation of temperature fields, it is often necessary to know the magnitude of the contact thermal resistance.

On the basis of a theoretical analysis and using experimental data for various materials and methods of machining, Shlykov [1] explained the main features of the relations governing contact thermal resistance in vacuo and in gaseous media and he derived equations which make it possible to calculate the magnitude of the contact thermal conductance as a function of various factors. Thus for contact heat exchange, he derived the following equation:

$$N_{\text{M}} = \frac{\alpha_{\text{M}} a}{\bar{\lambda}_{\text{M}}} = 0.32 \left(\frac{p}{3\sigma_{\text{B}}} \right)^{0.86}. \tag{1}$$

The radius of the contact spots was taken to be $4 \cdot 10^{-5}$ m based on analysis of the experiments; k is a coefficient depending on the average height of the microprojections of the roughness H_{av} of the contacting surfaces ($k = 1$ for $H_{av_1} + H_{av_2} > 30\ \mu$, $k = [30/(H_{av_1} + H_{av_2})]^{1/3}$ for $10 \le H_{av_1} + H_{av_2} \le 30\ \mu$; $k = 15/(H_{av_1} + H_{av_2})$ for $H_{av_1} + H_{av_2} < 10\ \mu$).

Equation 1 is convenient for engineering calculations since the single geometric characteristic of the surface which is needed for calculation is the mean height of the roughness microprojections.

However, it should be pointed out that the small number of experiments concerned with the region in which the values of the parameter $(p/3\sigma_B)k < 10^{-3}$, indicated a need for further experimental investigation of the contact heat exchange for low compressive loads (10^5 to 10^6 N/m^2), since for certain constructions, it is this range of loading which is of practical interest. The validity of extrapolation of Eq. (1) to the range of values $p/3\sigma_B < 10^{-3}$ requires experimental verification.

In this paper, we present the results of an experimental investigation of the contact thermal resistance of homogeneous and heterogeneous contact pairs (principally in vacuo) and we also compare these results with the dependence described by Eq. (1) and with certain other results.

The experiments were conducted on an apparatus for investigating contact thermal exchange consisting of a vacuum chamber, vacuum pumps, a lever system for applying a compressive load to the contact, balloons containing inert gases, regulating transformers, and measuring apparatus.

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TABLE 1. Characteristics of the Contact Surfaces of Experimental Specimens

Specimen material	Surface preparation method	Surface quality	Mean height of microprojections, μ
Kh18N9T	Machining	∇4	23,5
Kh18N9T	Machining	∇5	14,0
Kh18N9T	Machining	∇8	2,4
Kh18N9T	Grinding	∇8	2,2
Kh18N9T	Grinding	∇9	1,2
Kh18N9T	Machining followed by lapping	∇10	0,6÷0,8
Molybdenum	Grinding	∇9	1,0
Molybdenum	Grinding	∇9	1,07

The thermal current was generated with the help of an electric heater and a water condenser. The side surfaces of the samples were nearly adiabatic; this condition was attained with the help of screens and electric heaters for providing compensating heat. The temperature in the contact zone varied from 250 to 520°C in various experiments. Chromel–Alumel thermocouples having electrode diameter of 0.2 mm were used to measure the temperature. The ends of the thermocouples were brought out of the vacuum chamber through rubber gaskets with demountable flanges. The compressive load was applied with the help of a system of levers. The contact pressure was changed in the range 1.5 to $58 \cdot 10^5$ N/m^2. The experiments were conducted in vacuo with a pressure equal to 10^{-4} mm Hg. Some of the experiments were conducted in gaseous media with pressures of 1350 mm Hg.

The magnitude of the thermal current was determined from the temperature gradient between the measuring regions of the condenser and of the heater. The temperature drop across the contact was determined from reading four differential thermocouples, whose junctions were embedded in special ports in the sample at distances of 2 mm from the contact surface. The contact thermal resistance R_M was calculated from the equation

$$R_{M} = \frac{\Delta t}{q}.$$

The error in determining R_M was around 10%.

The experimental specimens were cylinders of 35 mm diameter and 20 mm height or else parallelepipeds of $43 \times 43 \times 20$ mm^3, made from Kh18N9T steel and molybdenum. The contact surfaces were prepared by machining, polishing and lapping. The surface quality was investigated with the help of a profilograph–profilometer of the "Kalibr" plant. Typical profilograms of the contact surface are shown in Fig. 1. The heights of the surface microprojections of various samples are given in Table 1.

Some experimental dependences of the thermal contact resistance on loading in vacuo are shown in Fig. 2. For some of the pairs, data for the same relationship obtained in a gaseous medium are included for comparison. In addition, the dependences of the contact thermal resistance on temperature in the contact zone were obtained for a constant load. The general character of these relationships agrees with other investigations of contact thermal exchange.

Figure 3 shows the results of a processing of these experimental data (the group of points corresponding to curves 13 through 16) in the form of the relationship of dimensionless parameters $N_M = \alpha_M a / \bar{\lambda}_M = f[(p/3\sigma_B)k]$, introduced by Shlykov [1]. We also include for comparison results obtained in [1] (points 1 through 8) and also the results of certain other investigations (9-12).

From the data of Fig. 3, it is evident that in the region where the parameter $(p/3\sigma_B)k < 10^{-3}$, a large number of points lie above curve I, and for higher values of the parameter $(p/3\sigma_B)k$ the experimental points of various authors are rather densely distributed around curve I. The existing scatter of experimental points is to a considerable degree determined by the experimental error associated with evaluation of α_M (5 to 20%), and also by the very approximate values of the mean height of the microprojections used in some investigations of contact thermal exchange. The physicomechanical properties of the surface layer of a given material can also differ because of differences in deformation of the surface layer resulting from the machining process and because of other reasons. In this respect, the generalized experimental data on contact thermal exchange in vacuum devised in [1] have a particularly important significance from the point of view of obtaining a relatively substantiated calculation formula.

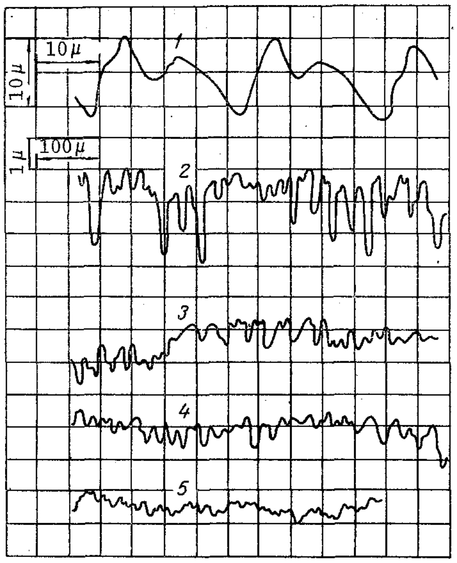

Fig. 1

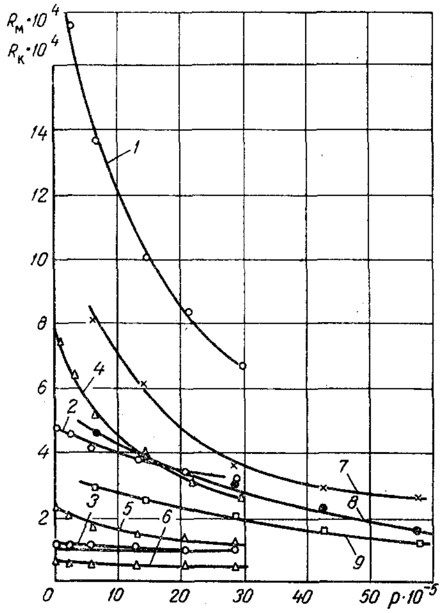

Fig. 2

Fig. 1. Typical profilograms of the contacting surfaces of experimental specimens: 1) Kh18N9T, machining ∇5; 2) Kh18N9T, grinding ∇8; 3) Kh18N9T, grinding ∇9; 4) molybdenum, grinding ∇9; 5) Kh18N9T, machining with subsequent lapping ∇10.

Fig. 2. Dependence of contact thermal resistance on compressive load at a constant temperature in the contact zone (R_M, $m^2 \cdot$deg/watt; p, N/m^2; R_K, $m^2 \cdot$deg/watt; R_M, curves 1, 4, 7, 8, 9; R_K, curves 2, 3, 5, 6): 1-3) molybdenum–Kh18-N9T steel, ∇9/∇5, in vacuo, argon and helium, respectively, t_K = 500°K; 4-6) molybdenum–Kh18N9T, ∇9/∇8, in vacuo, argon and helium, respectively, t_K = 500°C; 7-9) Kh18N9T–Kh18N9T, in vacuum; 7) ∇8/∇8, t_K = 470°C; 8) ∇9/∇9 t_K = 520°C; 9) ∇10/∇10, t_K = 490°C.

The approximating relation for experimental data in the range of parameter values $(p/3\sigma_B)k = 2 \cdot 10^{-4}$ to $8 \cdot 10^{-3}$ (92 experimental points for various materials and various methods of processing) can be written in the form

$$\frac{\alpha_M a}{\lambda_M} = c\left(\frac{p}{3\sigma_B}\right)^m. \tag{2}$$

A least mean squares fit to an expression of the type (2) yields the following expression:

$$\frac{\alpha_M a}{\lambda_M} = 0.118\left(\frac{p}{3\sigma_B}k\right)^{0,66}. \tag{3}$$

As the experimental data on contact thermal exchange in vacuo for small compressive loads accumulate, Eq. (3) can be made more accurate. In particular, it is necessary to investigate more thoroughly the effect of temperature on contact thermal resistance. In this connection, it was shown in [1], that Eq. (1) is valid for temperatures of the contact zone which are no more than 0.3 of the melting point of the material, and as the temperature increased, it was necessary to take creep into account since this phenomenon leads to an increase in contact thermal conductance. On the basis of the data presented in this paper, it is reasonable to conclude that as the temperature increases, the magnitude of the parameter $\alpha_M a/\overline{\lambda}_M$ measured experimentally is somewhat higher than that computed from Eqs. (1) and (3).

The nonlinearity of the relation $\alpha_M a/\overline{\lambda}_M = f[(p/3\sigma_B)k]$ in semilogarithmic coordinates for a wide range of values of the parameter $(p/3\sigma_B)k$ is also confirmed by a theoretical analysis of the contact thermal resistance of variously processed metal surfaces. We note first of all that the magnitude of the contact thermal resistance and the character of its dependence on load is determined to a considerable degree by the waviness of the surface. This can be demonstrated if we take into account the geometric characteristics of the processed surface as proposed in [13, 14]. The need to take surface waviness into account in a theoretical analysis of contact thermal exchange is demonstrated specifically by results of investigations of contact thermal exchange in gaseous media. Shlykov [1] showed that experimental data on thermal conductance of thin layers of the gaseous media in the contact region are fitted best by a relation in which the maximum gap in the contact region is assumed to have the value 2 $(H_{av_1} + H_{av_2})$. Meanwhile, a comparison of the geometric characteristics of the roughness and waviness given in [14] shows that for surface processing methods like machining, milling and grinding, the height of the waviness is approximately equal to the height of the roughness microprojections and, consequently, the maximum gap in the contact can be represented approximately by the sum $2H_{av_1} + 2H_{av_2}$.

Figure 4 shows in simplified form a diagram of the contact between rough, wavy surfaces. The actual surface contact is formed within the limits of the contour areas formed by the crumpled waves. The thermal resistance of the contact can be written as the sum

$$R_{\text{M}} = R_1 + R_2, \tag{4}$$

where R_1 is the thermal resistance arising from the confinement of the thermal current to parts of the contour areas; R_2 is the thermal resistance arising from the confinement of the thermal current to the path of the actual contact area found within the limits of the contour areas.

If it is assumed that the distribution of portions of contour areas over a normal surface and the distribution of patches of the actual contact over the contour areas are equal, then using the known values of a, r_b, η_1, and η_2 (Fig. 4), the thermal resistance of the contacts can be defined as

$$R_{\text{M}} = \frac{\pi r_b \psi_1}{2\bar{\lambda}_{\text{M}}\eta_1} + \frac{\pi a \psi_2}{2\bar{\lambda}_{\text{M}}\eta_1\eta_2}, \tag{5}$$

where ψ_1 and ψ_2 are coefficients which take into account the mutual interaction of discrete portions of the contact [9]. Their magnitudes can be determined from the following approximate equations:

$$\begin{aligned} \psi_1 &= 1 + F(\eta_1) = 1 - 1.40925\eta_1^{1/2} + 0.29591\eta_1^{3/2} + 0.05254\eta_1^{5/2}, \\ \psi_2 &= 1 + F(\eta_2) = 1 - 1.40925\eta_2^{1/2} + 0.29591\eta_2^{3/2} + 0.05254\eta_2^{5/2}, \end{aligned} \tag{6}$$

where $F(\eta_1) < 0$, $|F(\eta_1)| < 1$ and $F(\eta_2) < 0$, $|F(\eta_2)| < 1$.

In studies of the contact heat exchange, information on the waviness of the surface as a rule is not included; therefore, analysis of the contact thermal resistance requires the use of an idealized model based on a regular surface waviness. According to Kragel'skii [11] and Demkin [12], the waves are always elastically deformed, but the character of the deformation of microprojections can vary. If we use equations for a regular spherical waviness, we obtain the following relation for the plastic contact of the microprojections:

$$R_{\text{M}} = \frac{\pi}{\bar{\lambda}_{\text{M}}}\left(\frac{A\psi_1}{p^{1/3}} + \frac{B\psi_2}{p^{\frac{6\nu-1}{6\nu}}}\right), \tag{7}$$

where ν is a parameter of the bearing surface curve; A and B are coefficients which depend on the geometric characteristics of the roughness and waviness and mechanical properties of the materials of the solids in contact with each other.

In the case of elastic contact of the microprojections:

$$R_{\text{M}} = \frac{\pi}{\bar{\lambda}_{\text{M}}}\left(\frac{A\psi_1}{p^{1/3}} + \frac{c\psi_2}{p^{\frac{6\nu+1}{6\nu+3}}}\right). \tag{8}$$

In the case of a contact between two rough surfaces, the parameter ν can change in the range $\nu = 2$ to 6 [12]; consequently, the exponent for the specific compressive stress in Eqs. (7) and (8)

$$\frac{6\nu - 1}{6\nu} = 0.915\text{–}0.97 \text{ and } \frac{6\nu + 1}{6\nu + 3} = 0.865\text{–}0.95.$$

Improving the quality of the surface leads to an increase in η_2 [12] and a decrease in the values of ψ_2 and R_2. It was also shown in [13] that the ratio of the wavelength to its height and the radius of curvature of the top of the wave increase as the processing improves in quality; this leads to an increase of the relative area η_1 and to a decrease of ψ_1 and R_1. Consequently, both R_2 and R_1 decrease as the surface smoothness increases. By using the geometric characteristics of the roughness and waviness introduced in the course

TABLE 2. Comparison of Experimental Data (for contact pairs exhibiting lack of planeness) with Theoretical Results Derived from the Idealized Model (Calculation based on Eq. (13))

Contacting pair	$\zeta=\frac{pr_H}{E_M d}$	$\frac{\bar{\lambda}_M}{\alpha_M r_H}$ experimental values	$\frac{\bar{\lambda}_M}{\alpha_M r_H}$ calculation from Eq. (1)	Equivalent departure from planeness, μ	Quality of surface treatment
Molybdenum X18H9T steel	$3{,}7\cdot10^{-3}$	2,97	5,70	5	∇9
	$8{,}4\cdot10^{-3}$	2,20	3,74		
	$1{,}78\cdot10^{-2}$	1,62	2,47		
	$3{,}66\cdot10^{-2}$	1,05	1,48		
	$5{,}54\cdot10^{-2}$	0,842	1,025		
	$7{,}42\cdot10^{-2}$	0,692	0,84		
Molybdenum – molybdenum	$6{,}3\cdot10^{-3}$	2,86	4,44	4	∇9
	$1{,}46\cdot10^{-2}$	2,02	2,85		
	$3{,}515\cdot10^{-2}$	1,35	1,61		
	$5{,}57\cdot10^{-2}$	1,01	1,11		
	$3{,}14\cdot10^{-2}$	0,676	0,758		

of the investigations [13] and [14], it can be shown that the relation between the parameters R_1 and R_2 can change as the method of treatment changes; it turns out that in many cases in the practically encountered range of contact pressures $R_1 > R_2$.

The parameter α_M, defined by Eq. (7) can be written in the form

$$\alpha_M = \frac{1}{R_M} = \frac{\bar{\lambda}_M}{\pi A}\,\frac{p^{1/3}}{\psi_1}\,\frac{1}{1+\frac{B}{A}\,\frac{\psi_2}{\psi_1}\,\frac{1}{p^{\frac{4\nu-1}{6\nu}}}}. \tag{9}$$

Since $|F(\eta_1)| < 1$ in Eq. (6), the parameter ψ_1^{-1} can be written in the form of a series [15]:

$$\frac{1}{\psi_1} = \frac{1}{1+f(\eta_1)} = 1 - f(\eta_1) + f^2(\eta_1) - f^3(\eta_1) + \ldots. \tag{10}$$

In agreement with the equations for elastic contact in the case of spherical waviness, the relation of the parameters η_1 to load is given by the equation

$$\eta_1 = np^{2/3}, \tag{11}$$

where n is a coefficient depending on the geometric characteristics of the waviness and on the mechanical properties of the material. When Eqs. (10), (6), and (11) are taken into account, Eq. (9) can be written in the form

$$\alpha_M = \bar{\lambda}_M\left(D_1p^{1/3} + D_2p^{2/3} + D_3p + D_4p^{4/3} + \ldots\right)\frac{1}{1+\frac{B}{A}\,\frac{\psi_2}{\psi_1}\,\frac{1}{p^{\frac{4\nu-1}{6\nu}}}}, \tag{12}$$

where D_1, D_2, D_3 . . . are coefficients which do not depend on the load.

If $R_1 > R_2$, then $\frac{B}{A}\,\frac{\psi_2}{\psi_1}\,\frac{1}{p^{\frac{4\nu-1}{6\nu}}} < 1$. The form of Eq. (12) qualitatively explains the nonlinearity of the relation $\alpha_M a/\bar{\lambda}_M = f[(p/3\sigma_B)k]$ in logarithmic coordinates as a function of the parameter $(p/3\sigma_B)k$ over a wide range of values. This theoretical analysis is somewhat simplified. Under actual conditions, the number of contour areas of tangency can change with loading because of the various heights of different waves. It should also be noted that for loads close to zero (for a small number of microprojections in the contact area) Eq. (4) can be inaccurate.

The transfer of heat in the contact under vacuum conditions occurs not only by means of thermal conduction through the patches of the actual points of contact, but also because of radiation. However, calculations have shown that even for small values of the complex $(p/3\sigma_B)k$, i.e., for coarse surfaces and

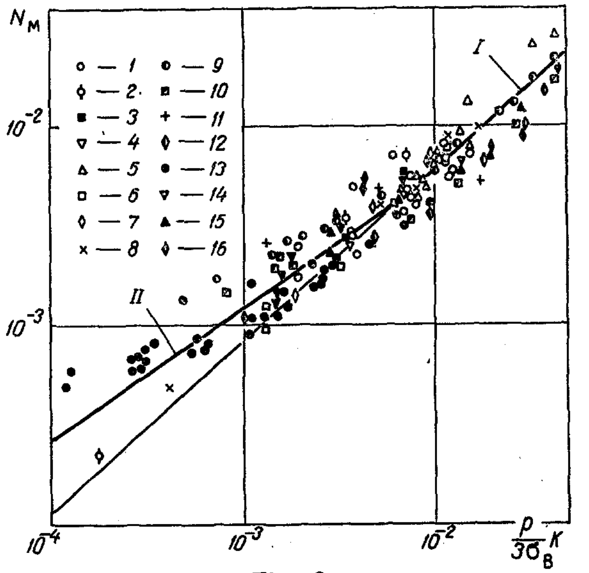

Fig. 3

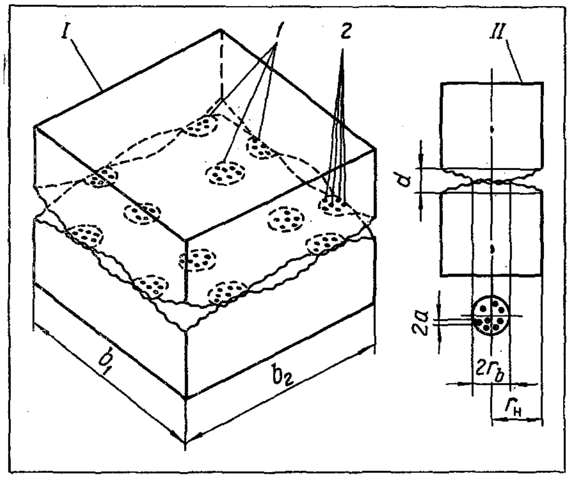

Fig. 4

Fig. 3. The results of processing experimental data related to contact thermal exchange in vacuo: 1) aluminum–uranium [5]; 2–4) aluminum–aluminum, aluminum–iron, and aluminum–uranium, respectively [3]; 5) uranium– "magnox" alloy [4]; 6) Kh18N9T steel [2]; 7) DT16 alloy [2]; 8) niobium [1]; I) approximate relationship (1), obtained from the data of 1-8; 9) steel 45, ∇5/∇5, ∇8/∇8 [7]; 10) steel 1 × 13, ∇8/∇8 [7]; 11) steel 20, ∇7/∇7 [8]; 12) magnesium ∇9/∇9 [6]; 13) molybdenum–Kh18N9T, ∇9/∇4, ∇9/∇5, ∇9/∇8; 14) Kh18N9T, machined, ∇8/∇8; 15) Kh18N9T, grinding ∇9/∇9; 16) Kh18N9T, machining with subsequent fine finishing, ∇10/∇10; II) approximation of Eq. (3).

Fig. 4. Sketch showing the contact between rough areas: 1) idealized sketch of the contact [1) part of the contour area of tangency S_1; 2) discrete patch area of the actual contact S_2; $b_1 \times b_2 = S_{nom}$; the nominal area of tangency; $\eta_1 = S_1/S_{nom}$ and $\eta_2 = S_2/S_1$ ratios of areas of tangency]; II) a contact element with one contour area in the center.

high hardness subjected to small compressive loads, the contribution of thermal radiation to thermal conductance of the contact does not exceed 3 to 5% at 500°C. The amount of heat transferred by radiation can be taken into account by using a linear approximation for the law governing radiative heat transfer between two parallel surfaces.

In conclusion, we want to focus on the following question. As is well known, processed surfaces can have not only roughness and waviness, but they can also possess a so-called macrogeometry characterizing certain errors in their shape which depend on many factors. The most characteristic macrogeometry is lack of planeness. The experimental investigations of Fried and Costello [10], of Clausing and Chao [9], and of other authors showed the strong effect resulting from lack of planeness on the contact thermal resistance. In tests performed by the author, a lack of planeness of 4 to 5 μ for a molybdenum–molybdenum pair, and a molybdenum–Kh18N9T steel with surface smoothness in the ninth class lead to a decrease in contact thermal conductance by a factor of 2 to 4 in comparison to the generalized relation (1). A detailed analysis of the effect of lack of planeness on the contact thermal resistance is beyond the scope of this paper. It should be noted, however, that to give a universal equation for calculating the dependence of contact thermal resistance on the extent of lack of planeness is not possible, since it would be necessary to know not only the magnitude but also the character of the lack of planeness. For surfaces which can be represented by the sketch shown in Fig. 4, II, Clausing and Chao give the following equation [9]:

$$\frac{\bar{\lambda}_M}{\alpha_M r_H} = \frac{2 \cdot 1.285 \zeta^{1/3}}{\pi \psi (1.285 \zeta^{1/3})}, \tag{13}$$

where r_H is the radius of the normal region of the contact (Fig. 4, II); $\xi = p r_H / \bar{E}_M d$ is a dimensionless quantity ($\bar{E}_M = (2E_{M_1} \cdot E_{M_2})/(E_{M_1} + E_{M_2})$ is the reduced elasticity modulus; d is the equivalent departure from planeness); $\psi\,(1.285\xi^{1/3})$ is a coefficient which takes into account the finite size of the thermal current channel [see (6)].

The departure from contact planeness investigated by the author was of such a character that the actual contact was formed around the periphery of the surface rather than at the center. Under these conditions, there can be several parts of the area of tangency, rather than a single such part; this was shown by Clausing and Chao [9]. This can explain the fact that the experimental values of contact thermal resistance are somewhat smaller than those given by calculation based on Eq. (13) (Table 2).

A comparison of Eq. (13) with experimental investigations by various authors shows that it is necessary to take into account thermal resistance caused by roughness which was neglected by the authors of [9]. This is especially important for small departures from planeness.

NOTATION

α_M	is the thermal conductivity of contact;
R_M	is the thermal resistance of contact;
λ_M	is the thermal conductivity of material;
$\bar{\lambda}_M = 2\lambda_{M_1}\lambda_{M_2}/(\lambda_{M_1} + \lambda_{M_2})$	is the reduced thermal conductivity (subscripts 1 and 2 refer to material of contacting bodies);
H_{av}	is the mean height of microroughnesses (subscripts 1 and 2 refer to contact surfaces);
k	is the coefficient depending on height of microroughnesses;
p	is the contact pressure (specific compressing load);
σ_B	is the ultimate strength (for materials with less strength);
E_M	is the elasticity modulus;
$E_M = 2E_{M_1}E_{M_2}/(E_{M_1} + E_{M_2})$	is the reduced elasticity modulus (subscripts 1 and 2 refer to material of contacting bodies);
S_{nom}, S_1, S_2	are the nominal, contour, and actual contact areas;
$\eta_1 = S_2/S_{nom}$, $\eta_2 = S_2/S_1$	are the relative contact areas;
a	is the mean radius of contact spot;
r_b	is the radius of contour area;
d, r_H	are the geometric parameters;
ψ_1, ψ_2	are the coefficients determined theoretically at given values of η_1 and η_2;
q	is the heat flux;
t_K	is the temperature in contact region;
Δt	is the temperature drop in contact;
R_K	is the thermal resistance of contact in gas media.

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